Lebanese American University **Spring 2006**

Byblos

Probability and Statistics **Date:** 14/06/2006

Final Exam **Duration:** 2h 30

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- The density function of the gamma distribution, with parameter α and β, is given by:

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{\frac{-x}{\beta}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

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1. (a) Find k such that $P(k < T < 3.250) = 0.985$ when $v = 9$.

   (b) Let X be a random variable which has a F-distribution with $v_1 = 11$ and $v_2 = 15$. Find x such that
$$P(X < x) = 0.01$$

2. During a laboratory experiment, the number of radioactive particles passing through a counter in 1 millisecond has a Poisson distribution with a mean of 4. What is the probability that more than 3 milliseconds are required before that 10 radioactive particles enter the counter?

3. A normal population with unknown variance has a mean of 32. We consider a random sample from this population of size 10. We calculate the sample mean and the sample variance if this random sample and we find 28 and 5, respectively. What is your conclusion?

4. A die is loaded in such a way that an even number is twice as likely to occur as an odd number. We consider the following experiment:

   "First we toss the loaded die. If we obtain an even number then we flip a coin three times, otherwise we flip the coin four times".

   We repeat the preceding experiment 128 times. Find the probability that we obtain exactly two heads

   (a) at least 50 times?

   (b) between 40 and 65 times inclusive?

   (c) exactly 55 times?

5. A mathematics placement test is given to all entering freshmen at a small college. A student who receives a grade below 35 is denied admission to the regular mathematics course and placed in a remedial class. The placement test scores and the final grades for 20 students who took the regular course were recorded as follows (x for the placement test score and y for the course grade):

x	y
50	53
35	41
35	61
40	56
55	68
65	36
35	11
60	70
90	79
35	59
90	54
80	91
60	48
60	71
60	71
40	47
55	53
65	57
50	70
50	68

A. We assume in this part that the distribution of the course grades is approximately normal.

   i. Find 95% confidence interval for the mean of the course grades.

   ii. Using 0.01 level of significance, test the hypothesis that $\mu_Y = 70$ against the alternative that $\mu_Y \neq 70$.

   iii. Find (an approximation of) the P-value for $\mu_0 = 70$, and interpret your result.

B. Now we will study the relation between the grade course and the placement test score.

   i. Find S_{xx}, S_{yy} and S_{xy}.

   ii. Find the fitted regression line $\hat{y} = a + bx$. Deduce the value of s^2.

   iii. Find 90% confidence interval for β in the regression line $\mu_{Y|x} = \alpha + \beta x$.

   iv. Find a 95% prediction interval for y_0 when $x_0 = 35$.

   v. Find the sample correlation coefficient r and interpret your answer.

   vi. Test the hypothesis that there is no linear association among the variables x and y for 0.1 level of significance.

   vii. Repeat the preceding question using 0.05 level of significance.

$\longrightarrow$

6. *Bonus question.* Let X be a continuous random variable such that its density function is

$$f(x) = \begin{cases} \dfrac{a}{x^4}, & x > 1 \\ \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Evaluate a.

(b) Now we consider the random variable Y defined by $Y = \inf\{2, X\}$. Find the cumulative distribution of Y (that is, find $P(Y \leq y)$ for $y \in \mathbb{R}$).

MARKS : 1. [15] **2.** [15] **3.** [15] **4.** [20] **5.** [35] Bonus question [10]

Lebanese American University **Summer I**

Byblos **2006**

Probability and Statistics **Date:** 15/09/2006

Final Exam **Duration:** 2h

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- The density function of the gamma distribution, with parameter α and β, is given by:

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{\frac{-x}{\beta}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

---

1. (a) Find k such that $P(k < T < -k) = 0.8$ when $v = 11$.
   (b) Find k such that $P(-1.108 < T < k) = 0.35$ when $v = 8$.
   (c) Find k such that $P(12.017 < \chi^2 < k) = 0.08$ when $v = 7$.

2. We consider two random variables X and Y such that:
   - X has a Gamma distribution with $\alpha = 5$ and $\beta = \dfrac{1}{3}$.
   - Y has a Gamma distribution with $\alpha = 6$ and $\beta = \dfrac{1}{2}$.
   - X and Y are independent.

   (a) We consider the random variable $Z = \max\{X, Y\}$. Find $P(Z < 4)$.
   (b) *Bonus question.* Repeat question (a) by replacing max by min.

3. A normal population has a standard deviation of 4. We consider a random sample from this population of size 9. We calculate the sample variance if this random sample and we find 36. What is your conclusion about the standard deviation of this population?

4. A die is loaded in such a way that an odd number is twice as likely to occur as an even number. We repeat the following experiment 108 times: **tossing the die three times**. Find the probability that we obtain an even sum
   (a) at most 50 times?
   (b) between 45 and 66 times inclusive?
   (c) exactly 64 times?

5. A study was made in a college to determine the relation between Mathematics grade and English grade. The following data were recorded (x for the Mathematics grade and y for the English grade):

x	y
70	74
92	84
80	63
74	87
65	78
83	90
70	70
94	75
55	65
66	90
45	55
98	75
55	75
60	71
60	85

A. We assume in this part that the distribution of the Mathematics grade is approximately normal.

   i. Find 90% confidence interval for the mean of the Mathematics grade.

   ii. Using 0.05 level of significance, test the hypothesis that $\sigma_X = 14$ against the alternative that $\sigma_X < 14$. Find (an approximation of) the P-value in this case and interpret your result.

B. Now we will study the relation between Mathematics grade and English grade.

   i. Find S_{xx}, S_{yy} and S_{xy}.

   ii. Find the fitted regression line $\hat{y} = a + bx$. Deduce the value of s^2.

   iii. Find 95% confidence interval for α in the regression line $\mu_{Y|x} = \alpha + \beta x$.

   iv. Find a 90% prediction interval for y_0 when $x_0 = 85$.

   v. Find the sample correlation coefficient r and interpret your answer.

   vi. Test the hypothesis that there is no linear association among the variables x and y for 0.05 level of significance.

MARKS : 1. [15] **2.** [15] **3.** [15] **4.** [20] **5.** [35] Bonus question [10]

Lebanese American University **Summer II**

Byblos **2006**

Probability and Statistics **Date:** 12/10/2006
Final Exam **Duration:** 2h

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- The density function of the gamma distribution, with parameter α and β, is given by:

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{\frac{-x}{\beta}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

---

1. (a) Find k such that $P(-k < T < k) = 0.95$ when $v = 5$.

   (b) Find k such that $P(k < T < 1.345) = 0.3$ when $v = 14$.

   (c) Let X be a random variable which has a F-distribution with degrees of freedom $v_1 = 10$ and v_2. We assume that

   $$P(X < 0.25) = 0.01.$$

   Find v_2.

2. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with standard deviation of 40 hours. How large a sample of light bulbs is needed if we wish to be 96% confident that our sample mean will be within 10 hours of the trues mean?

3. (a) We denote by Z a random variable which has a standard normal distribution. Find z such that $P(-z < Z < z) = 0.95$.

   (b) A normal population has mean of 20. We consider a random sample from this population of size 10. We calculate the sample mean and we find 18. What is your conclusion about the mean of this population if the standard deviation is known to be 3?

4. We consider the following experiment: **We toss two dice (one red and one blue). If we obtain an odd sum then we flip a coin 4 times. Otherwise, we flip the coin 3 times**.
   We consider the following event (denoted by A): "We obtain a number of heads equal to the number obtained on the red die".

   (a) Find $P(A)$. $\longrightarrow$

(b) Now we repeat the preceding experiment 192 times. Find the probability that the event A occurs

i. exactly 32 times?

ii. more than 20 times but at most 40 times?

iii. at least 25 times?

5. The following data represent the chemistry grades for a random sample of 15 freshmen at a certain college along with their scores on an intelligence test administered while they were still seniors in high school (x for the test score and y for the Chemistry grade):

x	y
65	85
50	74
55	76
70	87
65	94
55	82
50	76
65	80
70	90
70	80
55	70
60	80
65	70
60	70
60	85

A. We assume in this part that the distribution of the test score is approximately normal.

i. Find 95% confidence interval for the mean of the test score.

ii. Using 0.01 level of significance, test the hypothesis that $\sigma_X = 6$ against the alternative that $\sigma_X > 6$. Find (an approximation of) the P-value in this case and interpret your result.

B. Now we will study the relation between test score and Chemistry grade.

i. Find S_{xx}, S_{yy} and S_{xy}.

ii. Find the fitted regression line $\hat{y} = a + bx$. Deduce the value of s^2.

iii. Find 90% confidence interval for β in the regression line $\mu_{Y|x} = \alpha + \beta x$.

iv. Find a 95% prediction interval for y_0 when $x_0 = 70$.

v. Find the sample correlation coefficient r and interpret your answer.

vi. Test the hypothesis that there is no linear association among the variables x and y for 0.01 level of significance.

$\longrightarrow$

6. *Bonnus questions.* (The two questions are indepedent)

(a) Let X be a random variable which has a Gamma distribution with parameters $\alpha > 0$ and $\beta > 0$. We consider a random variable Y defines by $Y = aX$ where a is a positive real number. What is the probability distribution of Y?

(b) Find a constant k such that $P(3.921 - k < T < k) = 0.14$ when $v = 9$.

MARKS : 1. [15] **2.** [10] **3.** [15] **4.** [25] **5.** [35] Bonus question [10]

Lebanese American University **Spring 2007**

Byblos

Probability and Statistics **Date:** 16/06/2007

Final Exam **Duration:** 2h 30

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- The density function of the normal distribution, with parameter μ and σ, is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$

---

1. (a) Find k such that $P(-k < T < k) = 0.9$ when $v = 4$.
   (b) Find k such that $P(-2.359 < T < k) = 0.03$ when $v = 10$.

2. The probability that a person, living in a certain city, owns a car is estimated to be 0.8. Find the probability that the tenth person randomly interviewed in the city is the fourth one to own a car?

3. The number of customers arriving per hour at a certain automobile service facility is assumed to follow a Poisson distribution with mean $\lambda = 7$. What is the probability that more that one hour is required before three customers arrive?

4. Three manufacturers of car batteries claim that their batteries have an average life equals to 3 years. We select a random sample of size 5 from each manufacturer and we find the following data:

   - *First manufacturer*: The sample mean is 3.3 and the sample standard deviation is 0.25.
   - *Second manufacturer*: The sample mean is 3.2 and the sample standard deviation is 0.5.
   - *Third manufacturer*: The sample mean is 2.8 and the sample standard deviation is 0.2.

   From which manufacturer we must buy our batteries (the populations are assumed to be normal)?

5. A coin is loaded such that $P(\text{Tail}) = 2P(\text{Head})$. We consider the following experiment which consists of two parts:

   - First we flip the loaded coin several times to obtain head for the first time.

$\longrightarrow$

- Second we toss a die n times where n is the number of times that we flipped the coin in the first part.

We denote by A the following event: "The numbers obtained on the die in the second part of the experiment are all the same".

(a) Find $P(A)$.

(*Hint*: We recall that for $0 < a < 1$, we have $\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$).

(b) Now we repeat the preceding experiment 240 times. Find the probability that the event A occurs

i. exactly 85 times?

ii. more than 80 times but at most 100 times?

iii. at least 85 times?

6. The following data represent the Mathematics grades for a random sample of 15 freshmen at a certain college along with their scores on an intelligence test administered while they were still seniors in high school (x for the test score and y for the Mathematics grade):

x	y
65	85
50	74
55	76
70	87
65	94
55	82
50	76
65	80
70	90
70	80
55	70
60	80
65	70
60	70
60	85

A. We assume in this part that the distribution of the test score is approximately normal.

i. Find 95% confidence interval for the mean of the test score.

ii. Using 0.01 level of significance, test the hypothesis that $\sigma_X = 6$ against the alternative that $\sigma_X > 6$. Find (an approximation of) the P-value in this case and interpret your result.

B. Now we will study the relation between test score and Mathematics grade.

i. Find S_{xx}, S_{yy} and S_{xy}.

ii. Find the fitted regression line $\hat{y} = a + bx$. Deduce the value of s^2.

$\longrightarrow$

iii. Find a 90% prediction interval for y_0 when $x_0 = 80$.
iv. Find the sample correlation coefficient r and interpret your answer.
v. Test the hypothesis that there is no linear association among the variables x and y for 0.01 level of significance.

7. *Bonus question.* We say that a continuous random variable has a *lognormal* distribution with parameter μ and σ if its density function is given by

$$f(x) = \begin{cases} \dfrac{1}{\sigma x\sqrt{2\pi}} e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}}, & x \geq 0 \\ \\ 0, & \text{elsewhere} \end{cases}$$

(a) Prove that X has a lognormal distribution with parameter μ and σ if and only if the random variable $\ln X$ has a normal distribution with parameter μ and σ.
(b) Now we assume that X has a lognormal distribution with parameter $\mu = 2$ and $\sigma = 0.5$. Find $P(X > 4)$.

MARKS : 1. [10] **2.** [10] **3.** [10] **4.** [20] **5.** [20] **6.** [30]
Bonus question [10]

Lebanese American University **Summer II**

Byblos **2007**

Probability and Statistics **Date:** 12/09/2007

Final Exam **Duration:** 2h

Nota Bene. Use 4 digits in the fractional part of your answers.

1. (a) Find k such that $P(-2.069 < T < k) = 0.965$ when $v = 23$.

   (b) A random variable X has an F-distribution such that $v_2 = 2v_1$ and

   $$P(X < \frac{4}{9}) = 0.05$$

   Find v_1 and v_2.

2. It was estimated that 70% of the 17000 students of Louisiana State University are favor to an interdiction of smoking in the university campus. If 18 of these students are selected at random and asked their opinions, what is the probablility that more than 9 but less than 14 are favor to the interdiction?

3. An electrical component has a failure rate of once per 5 hours. What is the probability that 12 hours will elapse before 2 components fail?

4. (a) Explain the concept of large-sample confidence interval of the mean.

   (b) Three manufacturers of light bulbs claim that the average life of their bulbs is 900 hours. We select a random sample of size 49 from each manufacturer and we find the following data:

   - *First manufacturer*: The sample mean is 910 and the sample standard deviation is 46.
   - *Second manufacturer*: The sample mean is 907 and the sample standard deviation is 30.
   - *Third manufacturer*: The sample mean is 895 and the sample standard deviation is 25.

   From which manufacturer we must buy our light bulbs (remark that the populations **is not assumed to be** normal)?

5. We consider the following experiment: **We flip a coin several times to obtain two consecutive heads for the first time.** We denote by A the following event: "We flipped the coin at most six times".

   (a) Find $P(A)$.

   (b) Now we repeat the preceding experiment 64 times. Find the probability that the event A occurs: $\longrightarrow$

i. Exactly 50 times?
ii. More than 38 times but at most 48 times?
iii. At least 52 times?

6. The following data represents the content of 10 liters containers of a particular lubricant: 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. We assume the we have a normal population.

   (a) Find the sample variance.
   (b) Test the hypothesis $\sigma^2 = 0.03$ against the alternative hypothesis $\sigma^2 \neq 0.03$ using 0.01 level of significance.
   (c) Find the P-value in (b), and interpret your result.

7. The following data represents the weight (the random variable X in kg) and the chest size (the random variable Y in cm) of infants at birth:

x :	2.75	2.15	4.41	5.52	3.21	4.32	2.31	4.30	3.71
y :	29.5	26.3	32.2	36.5	27.2	27.7	28.3	30.3	28.7

   (a) Find S_{xx}, S_{yy} and S_{xy}.
   (b) Find the fitted regression line $\hat{y} = a + bx$. Deduce the value of s^2.
   (c) Find the sample correlation coefficient r and interpret your answer.
   (d) Test the hypothesis that there is no linear association among the variables x and y for 0.01 level of significance.

8. *Bonus question.* We recall that a statistic Θ is an unbiased estimator of a parameter θ if $E(\Theta) = \theta$. Prove that S^2 is an unbiased estimator of σ^2.

MARKS : 1. [10] **2.** [10] **3.** [10] **4.** [15] **5.** [15] **6.** [20] **7.** [20]
Bonus question [10]

Lebanese American University **Spring 2008**

Byblos

Probability and Statistics **Date:** 20/06/2008

Final Exam **Duration:** 2h

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- In all the probability questions of exercise 2, you must use a discrete or continuous probability distribution by giving its name.

1. (a) Find $P(-0.868 < T < 2.624)$ when $v = 14$.

   (b) Find k such that $P(k < T < 2.807) = 0.095$ when $v = 23$.

   (c) Find k such that $P(37.652 < \chi^2 < k) = 0.045$ when $v = 25$.

2. On average a certain intersection results in 2 traffic accidents per month.

   (a) Find the probability that at this intersection more than 1 accidents will occur:

      i. on a given month.

      ii. on 3 of the next 4 months.

      iii. for the second time in 2008 on May.

   (b) Find the probability that 3 months will elapse before 5 accidents occur.

3. (a) We denote by Z a random variable which has a standard normal distribution. Find z such that $P(-z < Z < z) = 0.97$.

   (b) A normal population has mean of 15. We consider a random sample from this population of size 9. We calculate the sample mean and we find 17. What is your conclusion about the mean of this population if the standard deviation is known to be 2?

4. We consider the following experiment: **We flip a coin several times to obtain HT for the first time (that is, a head follows by a tail for the first time).** We define the random variable X to be the number of times that we flipped the coin.

   A. i. What is the range of the random variable X?

      ii. Find the distribution of X, that is, find $P(X = n)$.

   B. Now we denote by A the event: "We flipped the coin at most four times".

      i. Prove that $P(A) = P(X \leq 4) = \dfrac{11}{16}$.

$\longrightarrow$

ii. Now we repeat our experiment 96 times. Find the probability that the event A occurs:

   a. Exactly 60 times?

   b. More than 55 times but at most 72 times?

   c. At least 58 times?

5. The following data represents the time in minutes required for 15 high school seniors to complete a standardized test: 30, 35, 28, 22, 38, 34, 26, 24, 30, 38, 23, 33, 34, 29 and 31. We assume that we have a normal population.

   (a) Find the sample variance.

   (b) Test the hypothesis $\sigma = 4$ against the alternative hypothesis $\sigma > 4$ using 0.025 level of significance.

   (c) Find the P-value in (b), and interpret your result.

6. The following data represents the grades of 9 students on a midterm report (x) and on the final examination (y):

x :	77	50	71	72	81	94	96	99	67
y :	82	66	78	34	47	85	99	99	68

   (a) Find S_{xx}, S_{yy} and S_{xy}.

   (b) Find the fitted regression line $\hat{y} = a + bx$. Deduce the value of s^2.

   (c) Estimate the final examination grade of a student who received a grade of 85 on the midterm report.

   (d) Find the sample correlation coefficient r and interpret your answer.

   (e) Test the hypothesis that there is no linear association among the variables x and y for 0.01 level of significance.

7. *Bonnus questions.* (The two questions are independent)

   (a) We recall that a statistic Θ is an unbiased estimator of a parameter θ if $E(\Theta) = \theta$. Prove that S^2 is an unbiased estimator of σ^2

   (b) Find a constant k such that $P(3.921 - k < T < k) = 0.14$ when $v = 9$.

MARKS : 1. [15] **2.** [20] **3.** [10] **4.** [20] **5.** [15] **6.** [20] **7.** [10]

Lebanese American University **Spring 2009**

Byblos

Probability and Statistics **Date:** 18/06/2009
Final Exam **Duration:** 2h

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- In all the probability questions of exercises 2, 3 and 4, you must use a discrete or continuous probability distribution by giving its name.

1. (a) Find k such that $P(k < T < 2.807) = 0.095$ when $v = 23$
   (b) Find k such that $P(k < \chi^2 < 22.760) = 0.7$ when $v = 18$.

2. On average, the number of injuries during a typical football game is 3. Find the probability that in a given football game more than 30 minutes will elapses before 2 injuries occur (we recall that the duration of a football game is 90 minutes).

3. A hospital has seven backup generators, and three are required to sustain the operations of the hospital upon blackout. If the probability that a given backup generator works is 0.8 and each of the backup generators are independent, what is the probability that at least three backup generators will be working if a blackout occurs?

4. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?

5. We consider two dice, one red and one blue. We assume that the red die is fair and that the blue die is loaded in such a way that odd number is twice as likely to occur as an even number. We toss the two dice 180 times. Find the probability that a total of 7 occurs more than 33 times but at most 41 times.

6. (a) We denote by Z a random variable which has the standard normal distribution. Find z such that $P(-z < Z < z) = 0.95$.
   (b) A normal population has mean of 20. We consider a random sample from this population of size 16. We calculate the sample mean and we find 21. What is your conclusion about the mean of this population if the standard deviation is known to be 3?

$\longrightarrow$

7. The following data represents the drying time, in hours, of a certain brand of latex paint: 3.4, 2.5, 4.8, 2.9, 3.6, 2.8, 3.3, 5.6, 3.7, 2.8, 4.4, 4.0, 5.2, 3.0, 4.8. We assume that we have a normal population.

   (a) Find the sample mean and the sample variance.

   (b) Using a 0.05 level of significance, can we claim that the true mean of the drying time is less than 4.

   (c) Find the P-value in (b), and interpret your result.

8. The following data represents the weight (x in kg) and the chest size (y in cm) of 9 infants at birth :

x :	2.75	2.15	4.41	5.52	3.21	4.32	2.31	4.30	3.71
y :	29.5	26.3	32.2	36.5	27.2	27.7	28.3	30.3	28.7

   (a) Find S_{xx}, S_{yy} and S_{xy}.

   (b) Find the fitted regression line $\hat{y} = a + bx$.

   (c) Estimate the chest size of an infant who has 4 kg as weight.

   (d) Find the sample correlation coefficient r and interpret your answer.

   (e) Test the hypothesis that there is no linear association among the variables X and Y using 0.05 level of significance.

9. *Bonus question.* Let X be a random variable such that $X \hookrightarrow g(x; p)$.

   (a) Find $P(X > n)$.

   (b) Prove that $P(X > n + m | X > m) = P(X > n)$ and interpret this result.

   (*Hint.* We recall that $\displaystyle\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$ for all $a \in]0, 1[$)

MARKS : 1. [10] **2.** [10] **3.** [10] **4.** [10] **5.** [10] **6.** [10] **7.** [20] **8.** [20] **9.** [10]

Lebanese American University | **Summer II**
Byblos | **2009**

Probability and Statistics | **Date:** 19/09/2009
Final Exam | **Duration:** 2h

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- In all the probability questions of exercises 2, 3, 4 and 5, you must use a discrete or continuous probability distribution by giving its name.

1. (a) Find k such that $P(-k < T < k) = 0.9$ when $v = 4$.
   (b) Find k such that $P(-2.359 < T < k) = 0.03$ when $v = 10$.

2. An interviewer needs to interview four people in a given street. What is the probability of having to ask eight shoppers in order to achieve this target?

3. An interviewer manages to question six respondents an hour on average.
   (a) What is the probability that in a period of twenty minutes she interviews no more than two people?
   (b) What is the probability that more than half hour will elapse before interviewing three people?

4. Pascal and Fermat are sitting in a café in Paris and decide to play a game of flipping a coin. If the coin comes up head, Fermat gets a point. If it comes up tail, Pascal gets a point. The first to get ten points wins the game. If the coin is loaded in such a way that $P(H) = 4P(T)$, find the probability that Pascal wins the game before Fermat gets 5 points?

5. A multiple choice quiz consists of 30 questions each with five possible answers from which only one is correct. A student passes the quiz if thirteen or more of his answers are correct.
   (a) What is the probability that a student who guesses blindly at all of the questions will pass the test?
   (b) What is the probability that the student passes the test if at every question he can eliminate three incorrect answers and then guesses between the remaining two?

$\longrightarrow$

6. (a) For $v = 5$, find k such that $P(-k < T < k) = 0.95$.
   (b) It is claimed that students at a certain university will score an average of 35 on a given test. Is the claim reasonable if a random sample of test scores from the university yields: 33, 42, 38, 37, 30 and 42? (we assume that the population is normal).

7. Air pollution is determined by measuring several different elements that can be detected in the air. One of them is carbon monoxide. The following sample of daily readings was obtained from a local newspaper: 3.5, 3.9, 2.8, 3.1, 3.1, 3.4, 4.8, 3.2, 2.5, 3.5, 4.4 and 3.1. We assume that we have a normal population.
   (a) Find the sample mean and the sample variance.
   (b) Does the sample show sufficient evidence to allow us to conclude that the carbon monoxide level is low, that is, $\mu < 3.9$ at $\alpha = 0.05$?
   (c) Find the P-value in (b), and interpret your result.

8. The following data represents the number of study hours spent by students outside of class for a course in statistics (x) and their scores in an examination given at the end (y):

x :	20	10	34	23	27	32	18	22
y :	64	61	84	70	88	92	72	77

   (a) Find S_{xx}, S_{yy} and S_{xy}.
   (b) Find the fitted regression line $\hat{y} = a + bx$.
   (c) Estimate the score of a student who studied 30 hours outside the class.
   (d) Find the sample correlation coefficient r and interpret your answer.
   (e) Test the hypothesis that there is no linear association among the variables X and Y using 0.05 level of significance.

9. *Bonus question.* To test the probability of the success p of a binomial population, we select a large size random sample and we use the following z-value:

$$z = \frac{\bar{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

If a standard medical cures about 70% of patients with a certain disease and a new medication cured 148 of the first 200 patients on whom it was tried, can we conclude that the new medication is better (use a 0.05 level of significance).

MARKS : 1. [10] **2.** [10] **3.** [10] **4.** [10] **5.** [10] **6.** [10] **7.** [20] **8.** [20] **9.** [10]

Lebanese American University **Spring 2010**

Byblos

Probability and Statistics **Date:** 08/06/2010

Final Exam **Duration:** 2h

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- In all the probability questions of exercises 2, 3, 4 and 5, you must use a discrete or continuous probability distribution by giving its name.

1. (a) Find k such that $P(-k < T < k) = 0.9$ when $v = 4$.

   (b) Find k such that $P(-2.359 < T < k) = 0.03$ when $v = 10$.

2. On average, the number of emails arriving to an office is 3 per hour. Find the probability that more than 20 minutes will elapses before 2 emails arrive.

3. In a process that manufactures bearings, 90% of the bearings meet the specification. A shipment contains 500 bearings. A shipment is acceptable if at least 440 of the 500 bearings meet the specification.

   (a) What is the probability that a given shipment is acceptable?

   (b) What is the probability that more than 285 out of 300 shipments are acceptable?

4. A family decides to have children until it has three children of the same gender. We assume that: $P(B) = 0.45$ and $P(G) = 0.55$. Let X be the number of children in the family.

   (a) Find the probability distribution $f(x)$ of X.

   (b) Find the mean and the standard deviation of X.

5. A tour operator has a small bus that can accommodate 18 tourists. The operator knows that tourists may not show up, so he sells 20 tickets. The probability that an individual tourist will not show up is 0.1, independent of all other tourists. Each ticket costs 50\$, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100\$ (ticket cost + 50\$ penalty) to the tourist. Let X be the gain of the tour operator (per bus). What is the expected revenue of the tour operator, that is, μ_X?

$\longrightarrow$

6. Suppose you have purchased a filling machine for candy bags that is supposed to fill each bag with 16 oz of candy. Assume that the weights of filled bags are approximately normally distributed. A random sample of 10 bags yields the following data (in oz): 15.87, 16.02, 15.78, 15.83, 15.69, 15.81, 16.04, 15.81, 15.92 and 16.10.
(For information: 1 oz = 0.028 kg)

   (a) Find the sample mean and the sample variance.

   (b) Does the sample show sufficient evidence to allow us to conclude that the mean of fill weight is actually less than 16 oz at $\alpha = 0.05$?

   (c) Find the P-value in (b), and interpret your result.

7. The following data represents tire pressure (in kPa) for the right and the left front tires on a sample of 10 automobiles (x for the left and y for the right):

x :	184	206	193	227	193	218	213	194	178	207
y :	185	203	200	213	196	221	216	198	180	210

   (a) Find S_{xx}, S_{yy} and S_{xy}.

   (b) Find the fitted regression line $\hat{y} = a + bx$.

   (c) Estimate the left tire pressure is the right one is known to be 200 kPa.

   (d) Find the sample correlation coefficient between the pressure in the right tire and the pressure in the left tire, and interpret your answer.

   (e) Can we conclude that the population correlation coefficient between the pressure in the right tire and the pressure in the left tire is greater than 0.9? (use 0.05 level of significance)

   (f) Can we conclude that the population correlation coefficient between the pressure in the right tire and the pressure in the left tire is positive? (use 0.05 level of significance)

8. *Bonus question.* To test the probability of the success p of a binomial population, we select a large size random sample and we use the following z-value:

$$z = \frac{\bar{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

In a survey of 500 resident in a certain town, 274 said they were opposed to constructing a new shopping mall. Can we conclude that more than half of the residents in this town are opposed to constructing a new shopping mall? (use a 0.05 level of significance)

MARKS : 1. [10] **2.** [10] **3.** [10] **4.** [15] **5.** [15] **6.** [20] **7.** [20] **8.** [10]

Lebanese American University **Summer II**

Byblos **2010**

Probability and Statistics **Date:** 07/09/2010
Final Exam **Duration:** 2h

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- In all the probability questions of exercises 2, 3, 4 and 5, you must use a discrete or continuous probability distribution by giving its name.

1. (a) Find k such that $P(-k < T < k) = 0.9$ when $v = 4$.
   (b) Find k such that $P(-2.359 < T < k) = 0.03$ when $v = 10$.

2. On average the number of hits on a certain website is 4 per minute. Find the probability that more than 30 second will elapse before 3 messages arrive.

3. In a process that manufactures bearings, 90% of the bearings meet the specification. A shipment contains 500 bearings. A shipment is acceptable if at least 440 of the 500 bearings meet the specification.
   (a) What is the probability that a given shipment is acceptable?
   (b) What is the probability that more than 285 out of 300 shipments are acceptable?

4. A store has 200 modems in its inventory, 75 coming from Source A and the remainder from Source B. Of the modems from Source A, 20% are defective. Of the modems from Source B, 4% are defective. Calculate the probability that exactly two out of a random sample of five modems from the stores inventory are defective.

5. An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. What is the expected benefit under this policy (that is, find μ_X where X is the benefit under this policy)?

6. Suppose you have purchased a filling machine for candy bags that is supposed to fill each bag with 16 oz of candy. Assume that the weights of filled bags are approximately normally distributed. A random sample of 10 bags yields the following data (in oz): 15.87, 16.02, 15.78, 15.83, 15.69, 15.81, 16.04, 15.81, 15.92 and 16.10.
   (For information: 1 oz = 0.028 kg)
   (a) Find the sample mean and the sample variance.
   (b) Does the sample show sufficient evidence to allow us to conclude that the mean of fill weight is actually less than 16 oz at $\alpha = 0.05$?
   (c) Find the P-value in (b), and interpret your result.

$\longrightarrow$

7. The following data represents tire pressure (in kPa) for the right and the left front tires on a sample of 10 automobiles (x for the left and y for the right):

x :	184	206	193	227	193	218	213	194	178	207
y :	185	203	200	213	196	221	216	198	180	210

(a) Find S_{xx}, S_{yy} and S_{xy}.

(b) Find the fitted regression line $\hat{y} = a + bx$.

(c) Estimate the left tire pressure is the right one is known to be 200 kPa.

(d) Find the sample correlation coefficient between the pressure in the right tire and the pressure in the left tire, and interpret your answer.

(e) Can we conclude that the population correlation coefficient between the pressure in the right tire and the pressure in the left tire is greater than 0.9? (use 0.05 level of significance)

(f) Can we conclude that the population correlation coefficient between the pressure in the right tire and the pressure in the left tire is positive? (use 0.05 level of significance)

8. *Bonus question.* To test the probability of the success p of a binomial population, we select a large size random sample and we use the following z-value:

$$z = \frac{\bar{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

In a survey of 500 resident in a certain town, 274 said they were opposed to constructing a new shopping mall. Can we conclude that more than half of the residents in this town are opposed to constructing a new shopping mall? (use a 0.05 level of significance)

MARKS : 1. [10] **2.** [10] **3.** [10] **4.** [15] **5.** [15] **6.** [20] **7.** [20] **8.** [10]

Lebanese American University **Spring 2011**

Byblos

Probability and Statistics **Date:** 01/06/2011

Final Exam **Duration:** 2h

Nota Bene.

- Use 4 digits in the fractional part of your answers.
- In all the dicrete probability questions of exercises 2, 3 and 4 you must use a discrete or continuous probability distribution by giving its name.

1. (a) Find k such that $P(-k < T < k) = 0.8$ when $v = 7$.
   (b) Find k such that $P(-2.069 < T < k) = 0.965$ when $v = 23$.
   (c) Find k such that $P(37.652 < \chi^2 < k) = 0.045$ when $v = 25$.

2. Suppose that the probability that any given person will believe a tale about the transgressions of a famous actress is 0.8. What is the probability that the sixth person to hear this tale is the fourth one to believe it?

3. A soft-drink machine is regulated so that the amount of drink dispensed is approximately normally distributed with a standard deviation equal to 1.5 deciliters. Determine how large a sample of drinks is needed if we wish to be 95% confident that our estimate will be within 0.1 deciliters of the true mean?

4. A company has 100 independent maintenance centers that received service calls. On average, the number of service calls received by each maintenance center is 6 per hours.
   (a) Find the probability that, for a given maintenance center, more than 30 minutes will elapse before receiving two calls?
   (b) Find the probability the number of maintenance centers for which more than 30 minutes will elapse before receiving two calls is more than 15 but at most 30?

5. A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean $\beta = 10$ years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made. Let X be the payment made under this insurance for one piece of equipment. At what level must x be set if the expected payment made under this insurance is to be 1000 (that is, find x such that $\mu_X = 1000$)?
   (*Hint*: The density function of the exponential distribution with parameter β is $f(x) = \frac{1}{\beta} e^{\frac{-x}{\beta}}$ if $x > 0$ and 0 elsewhere.)

$\longrightarrow$

6. The thickness of six pads designed for use in aircraft engine mounts were measured. The results, in mm, were: 40.93, 41.11, 41.47, 40.96, 40.80, and 41.32. Assume that these values come from a normal population.

   (a) Find the sample mean and the sample variance.

   (b) The target thickness is 41.2 mm. Can you conclude that the mean thickness differs from the target value at 0.05 level of significance?

   (c) Find the P-value in (b), and interpret the result.

7. For a sample of 12 trees, the volume of lumber x (in m^3) and the diameter y (in cm) at a fixed height above ground level was measured. The results were as follows::

x :	0.81	1.39	1.31	0.67	1.46	0.47	0.80	1.69	0.30	0.19	0.63	0.64
y :	35.1	48.4	47.9	35.3	47.3	26.4	33.8	45.3	25.2	28.5	30.1	30.0

   (a) Find S_{xx}, S_{yy} and S_{xy}.

   (b) Find the fitted regression line $\hat{y} = a + bx$.

   (c) Find 95% prediction interval for the diameter of a tree whose volume is 1 m^3.

   (d) Find the sample correlation coefficient between the volume of a tree and its diameter, and interpret your answer.

   (e) Can we conclude that the population correlation coefficient between the volume of a tree and its diameter is greater than 0.9 at 0.05 level of significance?

   (f) Find the P-value in (e), and interpret the result.

MARKS : 1. [15] **2.** [10] **3.** [10] **4.** [15] **5.** [15] **6.** [20] **7.** [25]